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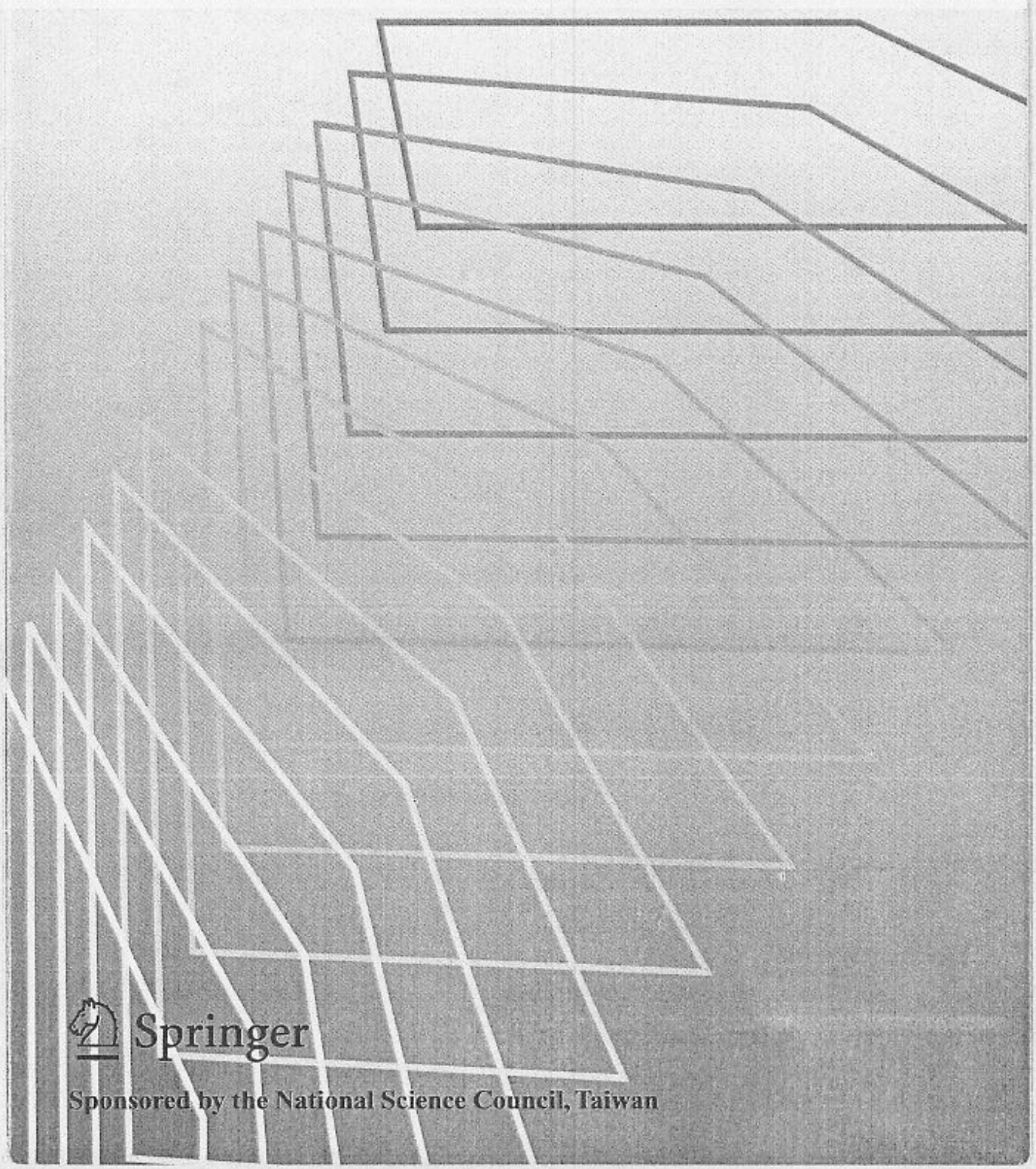

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PI-JEN LIN

USING RESEARCH-BASED VIDEO-CASES TO HELP PRE-SERVICE PRIMARY TEACHERS CONCEPTUALIZE A CONTEMPORARY VIEW OF MATHEMATICS TEACHING

ABSTRACT. The study inquired into the effect of research-based video-cases on pre-service teachers conceptualizing their understanding of contemporary mathematics teaching. The 43 participants enrolled in a Mathematics Method Course viewed and discussed 5 video-cases of primary teachers teaching. Journal entries, lesson plans, and microteaching observations were collected as data on which the following assertions were based. Pre-service teachers' responses to the 5 videos were more concerned with pedagogical content knowledge than mathematical content and students' learning. They refocused and deepened their awareness of students' learning and questioning skills through discussing the videos. The video-cases improved their construction of pedagogical representation and their ability to identify a problematic situation with multiple perspectives. These effects appeared to be influenced by the scaffold of three factors: vicarious experience to complement personal experience, watching and discussing video-cases enriched by the developers, and journal writing to foster deeper reflections.

KEY WORDS: mathematical journals, pre-service teachers, reflection, video-cases

Reacting to social pressures and drastic changes in the educational conditions of Taiwan, the Ministry of Education of Taiwan (MET) has issued two reforms dealing with mathematics education during the last two decades: the Curriculum Standards for School Mathematics (MET, 1993) and the Nine-Year School Curriculum (MET, 2001). The goals of these innovative curricula are aimed at the development of mathematical power such that students are able to explore, communicate, conjecture, reason logically, and use various mathematical methods effectively to solve non-routine problems. Students are encouraged to develop mathematical habits of mind, to read and discuss mathematics, and to build arguments about the validity of a conjecture.

BACKGROUND

The contemporary view of mathematics teaching rests on a constructivist perspective of learning that posits learning is contingent upon the activity and involvement of the learner. Cobb & Steffe (1983) stated, "teachers should continuously make a conscious attempt to see both their own and

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the children's actions from the children's point of view" (p. 85). This constructivist view of teaching emphasizes understanding and responding to individual students' experiences and needs instead of treating all students alike; on providing opportunities for student–teacher dialogue instead of reciting acquired knowledge and on supporting a classroom community with cooperation instead of competition. These reform documents define new roles for teachers related to the issues of communication, knowledge construction, supportive scaffolding, and reflection. Thus, many pre-service teachers are challenged by the discrepancy between the reform vision of mathematics instruction and their own learning experience with traditional pedagogy.

Helping pre-service teachers toward a contemporary view of teaching seems to require new experiences in learning mathematics that emphasize discourse in classrooms and require support from collaborative communities of practice. One of the ways to acquire such learning experiences in mathematics is by providing pre-service teachers with an arena for learning mathematics that actively engages in discourse. Frequently, pre-service teachers have not experienced such learning environments in their academic studies at university. Therefore, they need such experiences in their methods courses for teaching elementary school mathematics.

Some teacher educators suggest that learning based on others experiences and helping others learn from these experiences are effective ways to help teachers translate their theories into classroom practice (Carter, 1993; Lin, 2002; Richardson, 1993; Schifter, 1996a, 1996b). One way to implement these assumptions is through the use of *cases* that reflect aspects of classroom experience and raise issues (Harrington, 1995). Thus, the use of video-cases that illustrate others' experiences in encouraging students to participate in, and reflect on discourse centered on mathematical ideas, is the central pedagogical foundation for this study.

Mathematics Teaching Cases, Journal Writing, and Reflection

This study concentrated more on pre-service primary teachers' identification of the problems in case-teachers' teaching rather than on their own teaching. The reflection on the actions of others helped illustrate the variety of considerations related to primary school mathematics instruction. The understanding of pre-service teachers' capability to solve the problems identified in case-teachers' practice may be enhanced by asking them to suggest alternative solutions for improving the outcome.

Richert (1991) suggested that a curriculum of reflective teaching should be structured around authentic situations and social interactions. This implies the potential of using a video-case of actual teaching and focused

discussion for improving the reflective practice and the ability to reflect. Authentic situations supplemented with stories or narratives appear to have greater impact and are remembered. The process of viewing, analyzing, and identifying issues in the video-cases of authentic events not only provides individual experience for the novice teacher, but also generates new pedagogical perspectives by connecting the case with previous experience and other cases. Richert suggested that it was important to engage in social interactions with peers after watching and understanding the context of the video-cases. The cases allowed users to contribute their own ideas to group discussion and at the same time listen to alternative perspectives and learn from others. The use of cases appeared more likely to help users open their minds, become critical thinkers, and to focus on the possibility of alternatives.

Research on teacher education has demonstrated that journal writing is an effective means of generating reflective insights in order to make autonomous decisions (Castle & Aichele, 1994; Gould, 2000) and to promote conceptual change (Peters, 2000). In a similar way to how effective mathematics teaching helps learners improve their conceptual understanding pre-service teachers improve their understanding and practice as a result of experience and reflection. Conceptual change is closely related to reflection, e.g., the cognitive conflict that pre-service teachers experience may be the result of realizing that existing concepts are insufficient, when given new evidence. Cognitive conflict can motivate reflection, and the resulting resolution of the conflict can lead to conceptual change. The quality of reflection can be improved by developing pre-service teachers' abilities to observe accurately and discriminately and to view a situation from multiple perspectives (White, 1993).

The use of cases for teacher education has been advocated by both researchers and educators (Carter, 1993; Harrington, 1995; Richardson, 1993). Cases are vehicles for establishing a dialogic model of connecting theory and practice (Ball & Cohen, 1999; L.S. Shulman, 1992). Case-method instruction is performed for various purposes (Merseth, 1996). Cases can be exemplars to establish the best practice or to make the effective teaching more public and available for others to analyze and review (Sykes & Bird, 1992). Cases can be used to practise problem solving in which the cases portray problematic situations that require problem identification, analysis, and decision-making (Kleinfeld, 1991). The use of cases can be an effective means to develop habits and techniques for reflection (Kleinfeld, 1992; Lin, 2002). Results of these studies support the use of case-method instruction to help pre-service teachers acquire and improve their capabilities of reflection.

The effective use of cases involves case discussion. Research suggests that case discussion in a group setting plays a critical role in expanding and deepening pedagogical content knowledge (Barnett, 1991, 1998; Lin, 2002). Discussion about cases fosters personal reflection through an external process in which multiple perspectives and comments were shared (Lin, 2002; J.H. Shulman & Colbert, 1988). In case discussion, the issues of each case are centrally significant to teachers' focus and concerns and build toward a climax that exposes the dilemmas in teaching. Such discussion creates a disequilibrium for the users that lead to both assimilation and accommodation in their thinking (Levin, 1993). Therefore, discussion as an essential part of the cases involved in the study seemed likely to build multiple perspectives of mathematical teaching and learning for pre-service teachers.

The features of cases include concepts that events that are real, based on valid research, and initiate critical discussion by users (Merseth, 1996). The research-based video-cases in this study are characterized by not only these three features, but also by an additional three features (Lin, 2000, 2002). First, these cases were constructed by classroom teachers and the researcher during a three-year in-service teacher education program. The second feature of the research-based video-cases is that the instructor of the cases participated in the research context in which the cases were constructed. The third feature involved in this study is that each case provided vicarious experience for pre-service teachers and allowed them to foresee the complexity of the primary school classroom by observing case-teachers' realistic classroom situations.

Theoretical Perspectives

Teacher education has problems bridging the gap between theory and practice. One of the problems is the lack of compatibility between theoretical knowledge developed in academic settings and the situation teachers experience in classrooms. An approach in this study for linking theory and practice was adapted from Korthagen & Kessels' (1999) "Theory and theory". *Theory* aims to help us know more the formal knowledge applicable to broader situations, while *theory* – with a lower case 't' – developed by learners, aims to apply knowledge in a particular situation related to the context in which it is developed. In a traditional approach to teacher education, pre-service teachers learn general educational theories about schools, curriculum, and instruction and they then try to apply these ideas to what happens in classrooms. This approach from *Theory* to *theory* has not been overwhelmingly successful.

Kessels & Korthagen (1996) proposed a framework for teacher education focusing on a way to frame teacher knowledge and on the relation between teacher knowledge and behavior. The framework is based on four stages: creating suitable learning experiences, promoting further awareness and reflection, offering theoretical bases from empirical research, and encouraging student teachers to act in a productive manner. This framework presents the notion that the stories occurring in actual classrooms as the interpretation of an educational situation express pre-service teachers' knowledge or beliefs (L.S. Shulman, 1992). Thus, the use of video-cases with narrative supports seems likely to help pre-service teachers reflect on their knowledge, beliefs, and behavior.

METHOD

This case study utilized research-based video-cases in an attempt to explore a well-defined, confined system: a group of elementary pre-service teachers over time through detailed, in-depth data collection involving multiple sources of information. Case studies rely on multiple sources and triangulation to support claims about the specific context.

Participants

The participants of the study were 43 pre-service teachers attending a one-year Elementary Teacher Education Post-Baccalaureate Program at a teachers college. The 35 females and 8 males ranged in age from 23 to 35 years, including those who had recently graduated from a four-year university course without an education program. Thirty-four of these pre-service teachers had worked in private companies or government institutes for several years; 4 had been substitute teachers in elementary schools, and 5 had just received their bachelor's degree before enrolling in this program. All participants had the same purpose in that they were eager to become an elementary school teacher and were enrolled in the same course.

Setting

This study was designed to improve pre-service teachers' understanding of contemporary mathematics teaching and their reflective practice by using research-based video-cases. The cases were integrated into a course called the Methods for Teaching Elementary School Mathematics (Mathematics Methods Course). The weekly two-hour Mathematics Methods Course met for 32 hours during the semester and consisted of two kinds of activities, observing cases and microteaching.

First Part of the Course

The first part of the course was designed to increase pre-service teachers' awareness of student-centered approaches emphasized in the innovative curriculum via watching and discussing video-cases. Five video-cases were used as part of the activities of the course. Case 1 was to demonstrate the various semantic structures of word problems. Case 2 was to clarify students' representation as emphasized in the innovative curriculum. Case 3 (Figure 1) was to illustrate how a well-organized activity is able to promote third grade students' ability to use an advanced multiplication strategy $214 \times 4 = (\ )$. Case 4 was to demonstrate how the difficulties students have in transforming a fraction with a part-whole model to a fraction with iterating units. Case 5 was to illustrate the structural framework of measurement.

Small group discussions were used to broaden the pre-service teachers' perspectives of the central issues in the video-cases. The researcher, who was the course instructor, circulated among groups to listen to their concerns and facilitate their discussions. Afterwards, the pre-service teachers reported their concerns or issues to the whole class. Then, each case-teacher, the instructor in the video, was invited to participate in the discussion during the second hour of the course. When case-teachers participated in the discussions, the school principal assigned a substitute teacher for each case-teacher classroom. An honorarium for substitute teachers was provided by the research grant.

Second Part of the Course

The second part of the course was designed to understand how pre-service teachers put their understanding of mathematical teaching into practice. Eight hours over four weeks for microteaching, was designed to establish a new experience of teaching in an authentic classroom, for the pre-service teachers. Before microteaching, each group was asked to write an introductory lesson plan on the topic assigned by the case-teachers. The lesson plan was to consider how to use various examples and representations to promote students' thinking.

Microteaching involved pre-service teachers' teaching in a regular class divided into two small groups. Each group of five or six pre-service teachers was assigned to teach one 40-minute session in a small group. At least one person in each group did the teaching that the group had planned. The person who did the teaching was determined either by their willingness or by random selection. A person did not teach more than one session.

During the microteaching weeks, the first hour of a two-hour block was for microteaching and the second hour was a group discussion as well.

Objective : Enable students to use mathematical expressions to represent a multiplication word problems with three-digit multiplicahend and one-digit multiplier.

Instructional Context: The teacher posed two multiplication word problems.

Problem 1: Shi-Ming's mother bought 3 packages of dumpling in Carrifour. Each package costs NT$123, How much did Shi-Ming's mother need to pay for the dumpling? Using paper money and coins represents your thinking.

(a) Fu-Ann's solution

100 ⑩⑩ ①①①
100 ⑩⑩ ①①①
100 ⑩⑩ ①①①

(showing NT$123 for per package at a time)

(b) Wen-Kee's solution

100 ⑩⑩ ①①①
100 ⑩⑩ ①①①
100 ⑩⑩ ①①①

(showing 3 100 then 3⑩⑩ and 3①①①)

(1) The teacher (T) asked Fu-Ann and Wen-Kee to explain their thinking.

(2) T: Could you show us how you do this?

Fu-Ann: I showed the price of the first package 100 ⑩⑩ ①①① , then showed the price of the second package 100 ⑩⑩ ①①① , and the price of the third package 100 ⑩⑩①①①

T: How many hundreds did you show us? How many tens did you show us? How many ones did you show us?

Fu-Ann: There were 3 hundreds and 6 tens and 9 ones in my demonstration.

T: How did you do the 3 hundreds, 6 tens, and 9 ones?

Fu-Ann: 3 hundreds were the sum of a hundred for each package. 6 tens were the sum of 2 tens for each package, 9 ones were the sum of 3 dollars for each package.

(3) T: Can you show us how do you do?

Wen-Kee: I did 3 100 first, then 3 ⑩⑩ , and 3 ①①①.

T: How did you know next step you need to do with this?

Wen-Kee: 3 100 was resulted from a hundred in each 123. 6⑩ was resulted from 2 ⑩ in which 20 included in 123. 9① was resulted from 3 groups of 3 included in 123.

Problem 2: Shing Ming with his mom shopped in a supermarket together. They bought 4 packages of Lego. Each contains 214 pieces. How many pieces of Lego did they buy?

(a) Uei-Jee's solution

100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①

200 ×4=800
10 ×4=40
4 ×4=16
800+40+16=856

(b) Tein-Yui's solution

100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①

(c) Mei-Wein's solution

100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①
100 ⑩①①①①

200+200=400
400+200=600
600+200=800
4 ×10=40
4 ×4=16
800+40+16=856

(1) The teacher asked Tein-Yui to explain his thinking.

(2) T: How did you know you need to deal with 8 100 4 ⑩ , and 16① in the way?

Tein-Yui: There were NT$200 for each set of Lego. 4 sets cost NT$800, that was, 8 100 .

There was a ten included in 214. 40 was needed for 4 sets, so that the total was to be 4⑩.

There was a 4 included in 214, so that there were16 ones in total for 4 sets of Lego.

T: How did you write a mathematical expression to express 4 piles of 2 hundreds?

Tein-Yui: 200x4=800

T: How did you write a mathematical expression to express 4 piles of a ten?

Tein-Yui: 10x4=40

T: How did you write a mathematical expression to express 4 piles of 4 ones?

Tein-Yui: 4+4+4+4=16 or 4x4=16.

T: How much does the Lego cost?

Tein-Yui: 800+40+16=856. ….(To be continued)

Figure 1. Case 3: 214 × 4 = ( ).

TABLE I

The schedule of teaching and observing sessions in grade 1 or grade 4 between eight, groups of pre-service teachers in four microteaching weeks

	Microteaching			
Group	Week 1	Week 2	Week3	Week 4
Group 1	Teach grade 1-A [Whar time is it?]	Observe grade 4-A	Observe grade 1-A	Observe grade 4-A
Group 2	Teach grade 1-B [What time is it?]	Observe grade 4-B	Observe grade 1-B	Observe grade 4-B
Group 3	Observe grade 1-A	Teach grade 4-A [What are the properties of a shape?]	Observe grade 1-A	Observe grade 4-A
Group 4	Observe grade 1-B	Teach grade 4-B [What are the properties of a shape?]	Observe grade 1-B	Observe grade 4-B
Group 5	Observe grade 1-A	Observe grade 4-A	Teach grade 1-A [Which one is more?]	Observe grade 4-A
Group 6	Observe grade 1-B	Observe grade 4-B	Teach grade 1-B [Which one is more?]	Observe grade 4-B
Group 7	Observe grade 1-A	Observe grade 4-A	Observe grade 1-A	Teach grade 4-A [Division]
Group 8	Observe grade 1-B	Observe grade 4-B	Observe grade 1-B	Teach grade 4-B [Division]

Legend: [What time is it?] is the unit for microteaching session.

The schedule of the demonstration teaching and observing between the eight groups of pre-service teachers in the microteaching weeks is shown in Table I.

Case Discussion and Reflection

In the first stage of the study, the 43 pre-service teachers were randomly assigned to small groups for case discussion immediately following the viewing of a video-case. The discussion session was intended to encourage pre-service teachers to analyze teaching from multiple perspectives and to make sense of the teaching and learning they observed. In addition, the

case discussion was designed to identify the pre-service teachers' concerns for a mathematics teaching. Therefore, the course instructor started with the following questions to be answered in each case discussion:

- What concerns or problems would you like to address?
- What is, or were, your best solutions for the problems you identified?
- What did you learn from the case?

The course instructor did not provide supplementary information prior to the discussions but rather responded to the pre-service teachers' descriptions of the instructional event and the problems they chose to address.

Each pre-service teacher was asked to write an entry in a reflective journal after discussion to document individual concerns. It was assumed that the perspectives or issues the pre-service teachers described in their reflective journals were significant, challenging or of great interest to them. The issues written in the journals were integrated with the pre-service teachers' multiple perspectives or views, so that writing reflections in a journal is likely to improve their sensitivity to dealing with complex problems. After piloting the journal entries in the first two hours of the course, the course instructor provided the pre-service teachers with the reflective writings the case-teachers had written during the video-case constructed in such a way as to improve their skills in writing reflective entries. The researcher responded with written comments to the questions the pre-service teachers raised in their journals. One month later, the pre-service teachers were required to reread and mark the entries in relation to teaching, mathematics content, student learning, reflection, and their own conceptual change in order to promote in-depth reflection rather than simply replicate ideas from the prior discussion. Hence, the reflective journal not only served as a tool for improving the quality of reflection but also as a way of assessment of the course.

Data Collection and Analysis

Classroom observations, journal entries reflecting on the issues from the video-cases, microteaching, and discussions were the major data collected in the study. The practical knowledge expressed in the lesson plans for microteaching, their knowledge or beliefs demonstrated in the microteaching transcriptions, and the entries in the journal, provided triangulation of the target ideas. Due to space limitations, one of the four topics the pre-service teachers taught in the microteaching was selected to illustrate how they constructed pedagogical representations. The selected 40-minute geometry lesson in identifying a collection of geometric figures by its property was taught by Groups 3 and 4.

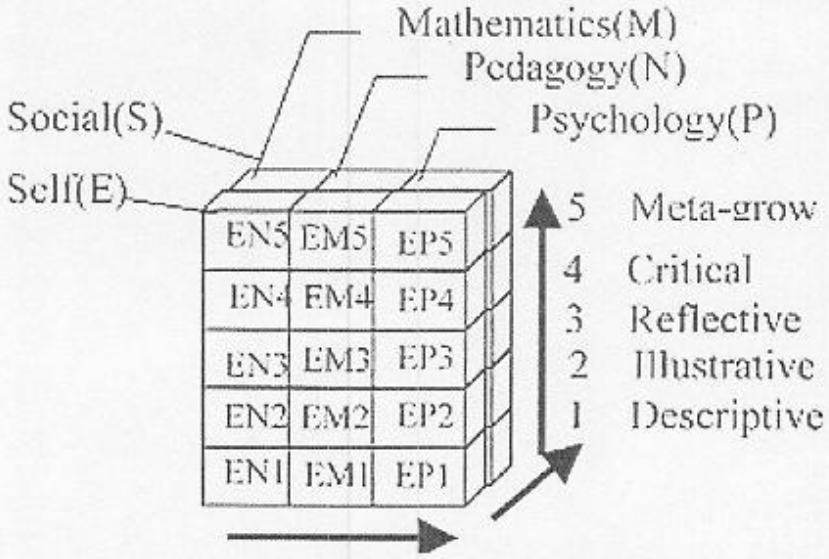

Figure 2. Schema of analyzing reflective journals.

A coding scheme was adapted from a system for analyzing reflective journals involving recursive reading of entries (Lin, 2001). Three dimensions emerged from this process: 'perspective,' 'view,' and 'level.' Perspective describes the ways participants organized and reviewed their teaching and learning experiences, focusing on the components of the mathematics content (M), pedagogy (N), or psychology (P). View describes whether the pre-service teachers reflected on themselves (E) or on others (S). Level indicates the degree of reflection and the way participants expressed themselves in writing: (1) descriptive, (2) illustrative, (3) reflective, (4) critical, and (5) meta-growth. A descriptive level involved only restating their own or others' opinions and an illustrative level journal entry was characterized by an explanation of statements. A reflective level involved individual experiences, by considering the meaning they held in terms of learning to teach and a critical level involved examining and comparing events from various perspectives. The meta-growth level personalized the findings of their analysis of their growth or others' conceptual change. The perspective, view, and level dimensions are depicted as a $3 \times 2 \times 5$ matrix (Figure 2).

Each cell of the matrix is represented by a letter and number code. EM1 means that the teachers wrote about their own experience (E) and reflected on the mathematics content (M) using a descriptive (1) mode of expression. Analysis began by coding the pre-service teachers' writing and an inductive search for patterns. An excerpt from a paragraph was coded as an example for illustration (Table II). Each code was counted, and frequencies were recorded.

TABLE II

Protocols of a paragraph encoded as an example

Protocols	Units of analysis	Codes	Frequencies of codes
Before discussing the case [3 + 4 = 7 vs. 4 + 3 = 7] ([ ] added) I would easily make the mistake [First graders could use 3 + 4 = 7 as frequent as 4 + 3 = 7 to express the problem, 'There were 4 kittens in the house, 3 more kittens came in the house. How many kittens in the house altogether'] ([ ] added).	Unit 1	EN3	
Although it does not mean that I am guaranteed to eliminate the mistake, I would take this [number sentence determined by semantic structure of addition word problems] ([ ] added) into account in my future teaching.	Unit 2	EN3	
From this case, I learned that I should pay attention to students' learning rather than from my own perspectives to help students to understand the meaning of the number sentence.	Unit 3	EP3	E = 3, N = 2, P = 1, level 3 = 3

Analysis: The paragraph was encoded into EN3, EN3, and EP3 by three units of analysis. The preservite teacher told about her past own experience or future (E) and reflected on pedagogy (N) and students' learning (P) with a reflective mode (3) of expression.

The coding schema was verified with the multiple analyses conducted by the researcher and two graduate students. Journal entries were coded separately and sequentially. The degree of agreement among the three raters was examined and the frequencies of codes for each paragraph were inspected. The coefficient of agreement among the raters was 0.84.

RESULTS

The main findings of the study were that pre-service teachers' responses to the video-cases stimulated insights into their thinking with respect to a more learner-centered philosophy. Their concerns – construction of pedagogical representation, and reflections on microteaching – were aligned with the assumptions of the new curriculum and contemporary view of mathematics teaching.

Pre-service Teachers' Concerns

The findings presented in this section were based on the analysis of the pre-service teachers' reflective journal entries. The pre-service teachers' concerns, raised after observing the five video-cases were classified into three domains: mathematics content, pedagogical content knowledge, and problems with students' learning (Table III). The mathematics content concerns were generally less frequent and specific to the nature of the math-

TABLE III

Concerns of pre-service teachers observing five video-cases

	Pre-service teachers' concerns Frequencies (%)				
Mathematics content (M)	Case I	Case II	Case III	Case IV	Case V
	69 (28%)	8 (3%)	28 (9%)	58 (17%)	53 (15%)
Pedagogical content knowledge (N)					
Ways of presenting a concept to students (NW)	43 (18%)	40 (13%)	30 (9%)	29 (9%)	54 (16%)
Teacher's role (NT)	29 (11%)	22 (7%)	32 (10%)	19 (6%)	10 (29%)
Grouping (NG)	10 (4%)	10 (3%)	3 (.5%)	8 (2%)	37 (11%)
Instructional strategy (NI)	5 (2%)	42 (13%)	66 (21%)	27 (8%)	33 (10%)
Reinforcement (NR)	30 (12%)	9 (3%)	4 (1%)	8 (2%)	6 (2%)
Physical arrangement (NP)	18 (7%)	61 (19%)	86 (27%)	74 (22%)	63 (18%)
Assessment (NA)	10 (4%)	27 (9%)	2 (.5%)	0 (0%)	4 (1%)
Total	145 (60%)	211 (68%)	223 (69%)	165 (49%)	207 (60%)
Students' learning (P)					
Students' characteristics (PP)	10 (4%)	2 (1%)	13 (4%)	15 (4%)	7 (2%)
Students' cognition of the topic (PC)	4 (2%)	60 (19%)	48 (15%)	79 (24%)	63 (18%)
Students' pre-experience (PE)	10 (4%)	7 (2%)	7 (2%)	11 (3%)	13 (4%)
Total	24 (10%)	69 (22%)	68 (21%)	105 (31%)	83 (24%)
Miscellaneous (M)	5 (2%)	27 (9%)	2 (1%)	8 (2%)	4 (1%)
Total	243 (100%)	315 (100%)	321 (100%)	336 (100%)	347 (100%)

ematics covered in each case. The pedagogical content knowledge concerns were the most frequent and included how to represent and formulate the subject matter to make it comprehensible to students. These concerns were classified into seven categories: (1) ways of presenting a concept to students, (2) teacher's role, (3) grouping, (4) instructional strategy, (5) reinforcement, (6) physical arrangement, and (7) assessment. The concerns about students' learning focused on students' characteristics, cognition of a topic, and prior experiences.

Mathematics Content

The mathematics content was identified as a concern more often in Cases 1, 4, and 5 (28%, 17%, 15%) than in Cases 2 and 3. The results indicated that the pre-service teachers were concerned about their knowledge of semantics in word problems, the fractional parts of continuous or discrete quantity, and the nature of measurements. It appeared that they were not aware of the importance of the semantic structures to first-graders' learning addition and subtraction. Compared with Case 1, in which 28% of the pre-service teachers had content concerns about "$3 + 4 = 7$ vs. $4 + 3 = 7$," the size and the complexity of the content involved in Case 2 was of little concern to the pre-service teachers: instead, most of their comments focused on the method of solving the problem presented in the example. For instance, they were more interested in discussing students' strategies for the problem that "Wei-Der has 34 candies, and then he ate 18 of them. How many candies does Wei-Der have now?" than the problem "Wei-Der has 39 candies, and then he ate 18 of them. How many candies does Wei-Der have now?" The former problem was solved by regrouping the numeral 34 for the subtracting of 18, while the latter problem was solved without regrouping.

Only 9% of journal entries were related to the fundamental meaning of multiplication. This result indicated Case 3 were identified as an easy topic. Fractions in Case 4 were identified as a difficult topic for the pre-service teachers. Pre-service teachers did not perceive a fraction as involving the part-whole model of continuous quantity and the set-subset model of discrete quantity. Moreover, they did not realize that 2/7 could be explained as either "2 *of 7 equal parts*" or "2 *pieces of* 1/7." The first interpretation can be explained as a part-whole model, but the second interpretation is based on a model of iterating units of fraction. The measurement of an angle introduced in Case 5 is different from other cases. A sizeable portion of the journal entries (15%) identified the curricular structure of angles as a concern. Their journals revealed that pre-service teachers were concerned about the use of the protractor, which focused on the structure of scales

as being quite different from their prior knowledge. Before discussing the video-case, they did not know about the developmental sequence and the curricular structure of measurement: using direct to indirect methods involving nonstandard and standard units. The findings showed that some of the cases helped the pre-service teachers to rethink the importance of dealing with the meaning of mathematics contents in the contemporary mathematics curriculum.

Pedagogical Content Knowledge

The pre-service teachers' responses to the five video-cases revealed more concerns about pedagogical content knowledge (60%, 68%, 69%, 49%, 60%) than the other two domains. The four highest concerns in this domain across the cases were physical arrangement (19%), ways of presenting a concept to students (13%), teacher's role (13%), and instructional strategy (11%).

Physical Arrangement. Concerns about the physical arrangement in the classroom addressed its organization in terms of the physical aspects of the learning arena. It included the spatial locations of the case-teachers and students in the video-case; for example, students' journal entries were exhibited on the classroom walls for students to learn mutually. Physical arrangement concerns also included concrete learning materials, such as *manipulates*, projectors, transparencies, pictures on the board, and cards with number sentences that were used in the classrooms.

The pre-service teachers' comments indicated that using *manipulatives* is an essential method to attract students' attention involving both physical arrangement and congnition. Their concerns in Case 2 about physical arrangement also considered students' cognition. The case-teacher gave students base-ten blocks to explain their solutions, so the pre-service teachers had to pay much attention to this part. A pre-service teacher (ID 8996067) commented in a descriptive mode of expression in her journal entries on how base-ten blocks affected students' thinking as follows.

> Some of the students solved the problem $39 - 18 = (\,)$ as two subtractions and one addition by $30 - 10 = 20$, $9 - 8 = 1$, $20 + 1 = 21$. The second graders further explained the mathematical expressions by removing 1 strip from 3 strips in which each strip represents 10, and removing 8 ones from 9 ones in which each one represents 1. This left 2 strips and 1 one representing 21. Meanwhile, some of the students solved the problems $39 - 18 = (\,)$ as progressive subtractions by $39 - 10 = 29$, $29 - 8 = 21$ and explained it by removing 1 strip from 3 strips and 9 ones. Then, 8 ones were removed from the 2 strips and 9 ones. (Journal, Video 2)

Case 3 is the only video-case using transparencies and a projector. The instructor wrote a student's problem on the transparency and posed it to

the class. A pre-service teacher (ID 8996080) outlined the advantages of using transparencies and a projector in her journal entry:

> There are two advantages of 'Posing a problem on the transparency': (1) Saving the time of writing a problem on the blackboard for students to solve, and (2) Saving the space on the blackboard for students to write. (Journal, Video 3)

Ways of Presenting a Concept to Students. Case 2 was intended to help pre-service teachers become aware of students' ways of thinking and to clarify the idea that manipulatives are a representation for explaining students' thinking instead of a tool. The use of manipulatives in the current curriculum is distinct from the former curriculum in that manipulatives are used to model the process of finding the answer instead of the demonstration of the answer. The use of manipulatives was a focus of attention during their discussion.

The pre-service teachers attended to the use of manipulatives for modeling students' thinking. In Case 3, the pre-service teachers viewed the case from the way students laid out the coins and bills and the instructor's pedagogical representation. Secondly, they made the distinctions in the order of demonstrating 214. They understood 200, 200, 200, 200 representing the meaning of 200×4, ⑩ ⑩ ⑩ ⑩ representing 10×4, and ①①①① ①①①① ①①①① ①①①① representing 4×4. Finally, the pre-service teachers complimented the case-teacher's pedagogical representation in making the connection between manipulatives and abstract mathematics expressions $200 \times 4 = 800$, $10 \times 4 = 40$, $4 \times 4 = 16$, $800 + 40 + 16 = 856$. On the contrary, in order to anchor students' understanding of the meaning of mathematics expressions, the case-teacher asked them to answer the following question:

> If a student solved the problem $214 \times 4 = (\ )$ by using $200 \times 4 = 800$, $10 \times 4 = 40$, $4 \times 4 = 16$, $800 + 40 + 16 = 856$, what did the mathematics expressions represent? (Journal, Video 3)

Comments like this indicate that these pre-service teachers realized that asking key questions to help students reflect on the meaning of number sentences and to promote advanced levels of thinking were more important than to merely demonstrate physical use of manipulatives.

Teacher's Role. The concerns regarding the teacher's role included enthusiasm, facial expression, pacing, speech, role, appearance, lesson preparation, and understanding of students' thinking. Enthusiasm refers to the teacher exhibiting a smile and a positive attitude. Pacing refers to the

teacher keeping the students moving cognitively with the lesson process. Role refers to the teacher as a director or facilitator of the learning process.

Case 2 helped the pre-service teachers to realize that a teacher should be a facilitator instead of a problem solver. A pre-service teacher (ID 8996058) stated in her journal entries:

> What I learned when I was young, on this topic was the algorithm of subtraction, I was told to efficiently compute $39 - 18 = (\ )$. Through the case discussion, I understood the role of the teacher was more than listening and sorting students' solutions into various categories. After students solved a problem, teachers had to provide them with the opportunity to clarify their thinking. The instructor is not permitted to talk too much instead of questioning students. (Journal, Video 2)

The case-teacher for Case 3 was a novice teacher. He was the only novice teacher among the five case-teachers. Thus, the concerns regarding the teacher's role for this case focused on him and his background. The case-teacher's background was quite similar to the pre-service teachers, and his teaching motivated the pre-service teachers to learn to teach more effectively.

Instructional Strategy. Concerns related to instructional strategy were about how the teacher leads the progression of the lesson. They included restating students' thoughts, asking students questions to facilitate their learning, and giving students feedback about their answers and the reasons for incorrect answers. Instructional strategies involved the ways teachers pose a problem for motivating students to learn.

The concerns about instructional strategies varied across video-cases. Questioning is a primary instructional strategy for student-centered approaches, but most of the pre-service teachers lacked the skill. The pre-service teachers drew heavily from the case-teacher's example in Case 3 for what questions to ask students. The key questions addressed in the writings of one pre-service teacher (ID 8996072) included:

- Why did you write this?
- What do 214 and 4 stand for?
- What does it mean by $214 \times 4 = (\ )$?
- How do you know to put 4 groups of 2 bills of a hundred dollars in a pile, to put 4 groups of ten-dollar bills [in another pile], and to put 16 groups of one-dollar bills [in the third pile]?
- Which of you did arrange the money with the order 8 hundred bills first, then 4 ten dollars and 16 one dollar?
- How much is 4 groups of 2 bills of a hundred?
- What is the relationship between 214×4 and $200 \times 4 = 800$, $10 \times 4 = 40$, $4 \times 4 = 16$, $800 + 40 + 16 = 856$?

Moreover, the following paragraph illustrates the pre-service teachers' reflection on questioning.

> Through case discussion, teachers' skill of questioning in dealing with the students explaining and solving problem could not be ignored. Asking students a key question beyond restating their solution was a vehicle to promote students to an advanced level. Yi-Shen's solution for solving $39 - 18 = (\,)$ was $30 - 10 = 20$, $9 - 8 = 1$, $20 + 1 = 21$. The key question to ask students could be "30 and 9 were not presented in the number sentence; where did you get them?" The questioning was able to provide the opportunity for students who used the same method to answer the question. Besides, the questioning was able to promote those who explained 39 as 39 ones toward an advanced level of treating 39 as a composition of 30 and 9. (Journal, Video 2)

Students' Learning. The pre-service teachers' concerns about students' learning included students' characteristics of learning, cognition of a specific topic, and prior experience (Table III). Students' cognition of a specific topic was somewhat variable (2–24%), but received the highest percentage concerns in the students' learning, while students' characteristics and prior experience were about equal.

Regarding students' characteristics of learning, in Case 3, a pre-service teacher (ID 8996051) noted why students heavily relied on counting paper money for understanding the meaning of $214 \times 4 = (\,)$. She explained that:

> The third-graders were motivated by attractive hands-on materials and had not mastered multiplication skills yet. To help third-graders move towards advanced multiplication strategy from iterating addition strategy ($214+214 = 428$, $428+214 = 642$, $642+214 = 856$), she [the teacher] suggested that the third-graders should be constrained to use mathematical expression of multiplication as the scaffold of solving the problem. She also mentioned that second-graders used the iterating addition strategy much more frequently than the multiplication strategy. (Journal, Video 3)

Case 5 involved students' difficulty with angles. The case illustrated students' difficulty with deciding if a rotated angle has the same number of degrees when it was labeled in two different positions. The case was intended to help pre-service teachers understand the way the case-teacher was correcting students' misconceptions about measuring an angle. Out of the five cases, concerns with students' prior experience were addressed most frequently in this case. A pre-service teacher (ID 8996054), while being a substitute teacher, commented in an illustrative way in his journal entries on students' difficulty with defining the concept of angles. He stated that:

> Students in the third grade started to learn the concept of angles embedded into a geometric shape. There was no opportunity for students to visualize how an angle was formed. The students I taught last year were asked to explain what an angle is. I was surprised by their responses. Their misconceptions included the width between two sides of the angle, the area formed by two sides, and the distance from the vertex to the label of the arrow.

However, I couldn't figure out how to help them. From the video, I found out that students should also be offered an experience of rotating the angle in which it was initiated by a starting line and rotated counterclockwise or clockwise ending in another position. (Journal, Video 5)

Concerns about students' cognition of a topic refer to the various strategies of problem solving and cognitive levels. The variability across the video-cases seems to indicate that the pre-service teachers had greater concerns about the significance of students' cognition in effective mathematics teaching as the course unfolded. Even though the effect of video-cases on pre-service teachers' conception of students' cognition cannot be supported by the frequencies of their concerns, it can be supported in the quality of their comments, such as identifying students' learning difficulties. The pre-service teachers made the distinction between students' solutions and the focus of the lesson that was on the process rather than simply the product described in the previous excerpts. These pre-service teachers appeared to experience quality change in conceptualizing students' cognition over the five video-cases. They became more able to anticipate the variety of strategies students would use to resolve a given problem. The pre-service teachers' concerns about learning reflected the philosophy underpinning the innovative curriculum dealing with a contemporary view of mathematics teaching,

The pre-service teachers had a short discussion to clarify the cognition underlying the two children's solutions in Case 2. Through the discussion, they realized the importance of understanding children's strategies for solving a problem. The strategy students use to solve a problem represents their ways of thinking. For instance, the cognitive level of those who explained that 39 stood for 3 tens and 9 ones in which tens and ones were used as two counting units at one time, was higher than that of those who interpreted 39 to be composed of 30 and 9 in which 30 and 9 were explained as subparts of 39. The pre-service teachers ranked students' solutions by cognitive levels during the discussion. A pre-service teacher (ID 8996083) reported that:

Yi-Shen's solution – in which 39 was decomposed into 30 and 9, and 18 divided into 10 and 8 – was readily connected to the algorithm of subtraction. Nevertheless, Wei-Ming, dividing 18 into 10 and 8, was able to deal with a greater number. (Journal, Video 2)

Another pre-service teacher (ID 8996063) described what she learned from the discussion in her journal:

Through the case discussion, I finally realized the reason why we needed to analyze and distinguish the difference between the two students' solutions. The importance of understanding students' various solutions was not for judging its value but for recognizing students' thinking. It would help me to identify which strategy belongs to which cognitive

levels and then help students to learn mathematics effectively when I start to teach after my graduation. (Journal, Video 2)

Overall, the data of Table III reveal that the total of frequencies of pre-service teachers' concerns across the dimensions from Case 1 to Case 5 was increasing (243, 315, 321, 336, and 347). This trend indicated that the pre-service teachers were able to describe a case with more sensitivity to subtle features of mathematics teaching as the course unfolded.

Pre-Service Teachers' Constructions of Pedagogical Representation and Reflections on School Practice

In addition to expanding and deepening the pre-service teachers' concerns, it is evident that the research-based video-cases improved their construction of pedagogical representation and reflections on microteaching. The pre-service teachers' pedagogical representations were collected from lessons plan, microteaching, and mathematical journals based on their microteaching. Their abilities to construct representations for teaching and reflections on microteaching were by direct applications of their concerns developed from the first part of the Mathematics Method course. The reflections on their microteaching frequently involved identifying an event to be problematic from expanded multiple perspectives.

Constructing Representations for Teaching

Overall, the lesson plans prepared by the pre-service teachers included learning goals, prior knowledge/skill, materials needed, and activities. In each activity of a lesson plan, they wrote more detail on anticipating students' solution strategies than those regularly provided in the textbook (Appendix: Sample lesson plan from Group 3).

The pre-service teachers in Group 3 realized that students in early fourth grade were expected to develop an advanced thinking level about geometric shapes and the ability to classify shapes in terms of their properties. The two activities were designed for students to develop the properties of parallelograms that go beyond the identification of a figure by its appearance. Nevertheless, they were not explicitly aware of the features characterized by each thinking level; but the pre-service teachers anticipated that students would sort a collection of geometric figures by a shape's appearance, name, or by thinking about properties (e.g., by sides either parallel or equal). They predicted that students would sort a collection of quadrilaterals into three categories:

Figure 3.

(1) By sides if parallel – two pairs of parallel sides (parallelograms, rectangles, squares, rhombus), one pair of parallel sides (trapezoids), or no parallel sides (kites);

(2) By sides if equal – four equal sides (squares, rhombus), two sets of equal opposite sides (parallelograms, rectangles), no equal sides (kites, trapezoids); and

(3) By name – parallelograms, rhombus, squares, rectangles, trapezoids.

However, analysis of the microteaching revealed that students gave one more categories involving the number of right angles; e.g., a square has four right angles. The figures with no right angles included parallelograms, kites, trapezoids, and rhombus.

Another aspect of constructing a representation of teaching was related to identifying students' misconceptions, errors, or difficulties. The pre-service teachers in Group 3 learned from their teaching about the students' difficulty with geometric figures in different orientations. They were surprised that students had difficulty identifying a rhombus in different orientations when compared to a square or a parallelogram that was rotated into an oblique orientation. As observed, a pre-service teacher moved a rectangle that was constructed by two pairs of equal straws into two pairs of parallel sides, demonstrated as Figure 3, and asked students "What is the new shape after moving?" Students responded: "a rhombus."

As well, the fourth graders had difficulty with identifying subclass relations, e.g., a square is not a rhombus. The pre-service teachers did not know of any instructional strategies to help students overcome these difficulties. During the group discussion immediately following the microteaching, the case-teacher suggested that it is not appropriate for fourth graders to learn the subclass relations and said that it is difficult to accept placement of squares in the rhombus group, since judgment about shapes is made on the basis of properties such as four congruent sides.

Ability to Identify a Problematic Situation from Multiple Perspectives. The observers of Group 3's microteaching session were able to identify problematic situations from multiple perspectives developed during the first part of the course. Two pre-service teachers from Group 5 responding to the sorting activities addressed the improper use of hands-on materials in the discussion and in their journals critically reflected on the instructor's questioning skill and identified a teaching situation to be problematic:

In the teaching of Group 3, they used the [drinking] straws to demonstrate a transformation of [a rectangle into a parallelogram] and [a square into a rhombus]. Besides, they also used a set of cards for motivating students' learning. [There were too many] cards to focus on the goal of the sorting activity. Prior to the instruction, the members of the group were not [aware] that the cards could be sorted repeatedly. As a result, the students were restricted to sorting the cards by only the given names: rectangle, parallelogram, trapezoid, rhombus, and kite. In fact, the cards could be sorted by their attributes. For instance, a square card can be put into either the square class or the rectangle class. Finally, it is hard for students to establish the subclass relations of shapes from this teaching. (ID 8996092, Meeting of Microteaching 1)

I did not know about the teaching of geometric figures and students' thinking level in learning the geometric topics ... until Ming-Fa [*the instructor of Group 3*] and Yo-Ling [*the case-teacher*] introduced the sequence of the geometric unit in the discussion right after the instruction: naming by its appearance, identifying its properties, relating to the relationship between two properties, and identifying subclass relations. However, in Group 3's instruction, I noticed that at the beginning of the activity 2 Ming-Fa wanted the students to sort the cards with various figures into different categories, but students were not given any clear clue. Consequently, students were confused with the criteria for sorting. They are suspect if the category can be done either by shape, sides, or by angle. At the very beginning of the sorting activity, [it was hard] for the fourth graders to get started. (ID 8996052, Journal of Microteaching 1)

DISCUSSION

The central issue of the study was to identify and verify an instructional scaffold that pre-service teachers could use to conceptualize contemporary views of mathematics teaching. The first stage included watching research-based video-cases and the second stage of the scaffold included observing case-teachers' teaching, preparing a lesson, and teaching the prepared lesson. The scaffolding in this teacher education program involves the following factors: vicarious experience to complement personal experience, watching video-cases enriched by the developers and case discussions to stimulate pre-service teachers' reflection, and journal writings to foster deeper reflection.

The pre-service teachers' perspectives of the standards-based teaching were achieved by using research-based video-cases to complement their learning experiences with mathematics. The video-cases illustrated case-teachers' experiences with encouraging students to participate in and reflect on discourse centered on mathematical ideas. The pre-service teachers were provided opportunities to apply their new perspectives about mathematical teaching to school practices. The practical knowledge of teaching constructed during the second stage served as a reference for veri-

fying their beliefs and the ideas learned through the course work. Thus, the course work for this preparatory program that encouraged pre-service teachers to question their dispositions and beliefs, is different from teacher education course work in traditional programs (Richardson, 1993).

Second, the research-based video-cases were a fundamental characteristic of this study. The distinction of the research-based video-case referred to in this study, from those described in the literature, was that the case-teachers and the researcher were involved in the original case construction and were readily able to offer a broader and more in-depth background to the cases during discussions. The case-teachers' participation enriched the information about instructional context, desired objectives, and status of the lesson. Consequently, the pre-service teachers' discussion of the video-cases did not require them to make questionable assumptions about these cases. Thus, the use of video-cases is an effective instructional method to help the pre-service teachers to translate their theory into classroom practice, since they made informed decisions when they faced similar problems in their actual teaching. Moreover, the teaching in the video-cases were situated in a real context that helped the pre-service teachers resolve problems they encountered in a specific classroom. This finding is consistent with the problem-solving literature suggesting that the problem solver who can identify a problem when viewing a practical instance will be able to identify the same problem when viewing a real instance (Schon, 1983, 1987).

The study indicated that the video-cases apparently influenced the pre-service teachers' concerns and improved the quality of their concerns about students' cognition but did not increase the frequency of their concerns about students' cognition. However, overall, the video-cases motivated pre-service teachers to rethink the importance of a student-oriented approach and to emphasize the need for engaging students with challenging mathematical tasks. For instance, Case 4 helped the pre-service teachers understand the significance of the students' cognition when they noticed the students' difficulty with a fraction and various strategies for resolving a problem. The pre-service teachers gained insights for student-centered teaching from the case-teachers in the study. The result of this study was not consistent with the assertion that novice teachers were concerned with mere survival rather than with student-oriented concerns (Brown, 1992). These pre-service teachers exhibited an ability to anticipate possible solutions, articulate students' difficulty with a specific topic, and identify problematic situations from multiple perspectives when they constructed their pedagogical representations. These outcomes appear to result from the viewing of video-cases and discussion.

Third, the results of pre-service teachers' microteaching indicated that the video-cases allowed the pre-service teachers to develop some ideas about the ways to solve real situated problems. This result is consistent with an earlier study that reflection on the actions of others helped illustrate the variety of considerations related to mathematics instruction (Harrington, 1995). In addition, the cases allowed users to contribute their own ideas to group discussion and at the same time listen to alternative perspectives and learn from others. The social interaction stimulated by the video-case discussion appeared to be the source of changes in the pre-service teachers' ways of thinking and in the breadth and depth of their pedagogical content knowledge. The pre-service teachers came to have open minds: became critical thinkers, and attended to the possibility of alternatives. This finding suggests that individual development appears first on the social plane followed by the psychological plane (Vygotsky, 1978). The pre-service teachers began to change their views of teaching from having a passive attitude toward the innovative curriculum and then move to challenge the reform.

Finally, journal writing helped the pre-service teachers learn to reflect in a more efficient way. However, it was found that the writings of the pre-service teachers involving the levels of reflections (descriptive, illustrative, and reflective modes) were not as deep as that of in-service teachers (Lin, 2001). A possible reason is that reflective writing was a new experience for them. However, the quality of reflection on students' learning for some pre-service teachers in the preparation program was improved by observing the videos and by asking them to offer alternatives for the problems identified in the video-cases. The results support other research findings (White, 1993).

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APPENDIX. LESSON PLAN

Grade level: Early fourth grade.

Objective: To establish the relationships among components of parallelogram.

Prior knowledge/skill:

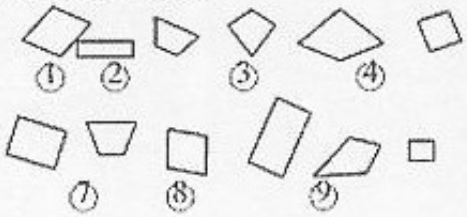
- Students need to be able to know the components of a triangle and quads.
- Students need to be able to identify a set of parallel lines and a set of perpendicular lines.
- Students need to be able to understand the properties of rectangles and squares.

Materials needed: Straws, pins, adhesive tape, plasticine, six sets of parallelogram with the size and the shapes of trapezoid, rhombus, and kites cards.

Activity 1: To name various quadrilaterals consisting of parallelogram, trapezoid, rhombus, and kites.

Teachers' key questions	Predicted students' reponses
1. What are the shapes by using two pairs of straws?	– Rectangles.
2. How do you know?	– It has four right angles.
3. Anything else?	– Two pairs of equal opposite sides.
4. If we have a movement for rectangle like  what is the new shape now?	Parallelograms
5.  Is this still a rectangle?	No.
6. What's the name of each shape?  ① ② ③	①-trapezoid. ②-rhombus. ③-kites.

Activity 2: To recognize the relationship among components of various quadrilaterals by sorting.

Teachers' key questions	Predicting students' refuses
1. Each group has a collection of cardboard cut-out polygons. The figures include: 	(Working in small group)
2. Sorting them into different categories.	Sol. 1: (4, 5, 11), (3, 8), (1, 6, 9, 2, 7, 10, 12) Sol. 2: (3, 8), (1, 9), (6), (4, 5, 11), (2, 7, 10, 12) Sol. 3: (3, 8, 4, 5, 11), (1, 6, 9), (2, 7, 10, 12)
4. Why did you put them in groups which belong together?	Sol. 1: No parallel sides, one pair of parallel sides, two pairs of parallel sides, respectively. Sol. 2: They arc trapezoids, rhombus, squares, kites, parallelograms, and rectangles, respectively. Sol. 3: No equal sides, two pairs of equal sides, four equal sides, respectively.
5. Why would you like to sort the cards by trapezoids, rhombus, squares, kites, and rectangles?	– Square has four equal sides and four right angles. – Rectangle has two pairs of equal sides and four right angles. – Rhombus has four equal sides. – Kite has one side equal to next to side. – Trapezoid has one pair of parallel sides.

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Department of Mathematics Education,
National Hsin-Chu University of Education,
521 Nan-Dah Road,
Hsin-Chu 300,
Taiwan
E-mail: linpj@mail.nhctc.edu.tw